Space Elevator

Dynamics Reference Manual

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in behalf of
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Contains Section 1 Only

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INTRODUCTION

This manual is a preliminary work aimed at providing a source of convenient dynamics-related information for those involved with all aspects of space elevator (SE) development and design. For some, this manual may serve as a primer of SE dynamics, for all, it is a source of specific constants, attributes and SE behaviors. This manual is a work in progress as much remains to be addressed as work proceeds on the project. Each section of the manual addresses a special aspect of information pertaining to the SE.

The dynamics attributes have been mostly derived from the time-domain simulation called GTOSS (Generalized Tethered Object Simulation). An outline of GTOSS is included (in Appendices A through D, etc) to allow the user to assess the pertinence of this simulation in providing such results for each aspect of SE dynamics. Related materials s re-organized and derived from these studies by the author appear in the papers listed among the references.

General Note*: Some items below should be included for future efforts*

a. Thermal response
b. Climber attitude dynamics
c. Ocean wave effects on longitudinal dynamics
d. Sun-Moon tidal effects
e. Aerodynamic pull-down response
f. Breakage debris-footprints
g. General pull-down response

1.0 GENERAL DATA, PARAMETERS, FORMULAS

1.1 USEFUL NUMERICAL CONSTANTS

This section addresses physical constants typically used in SE design calculations; these are broken down into physical constants, and broadly dynamics-related attributes.

1.1.1 General Physical Constants

Below are physical constants frequently used in SE calculations. These data are presented in metric and English units.

Nom. Ribbon Length	= **100,000 km = 10^7 m**
	= 328,083,985 = 3.28083985×10^8 ft
Nom. Ballast Mass	= **634,279.7 kg**
	= 43,462 slugs
	= 1,398,348.5 lbm
Earth Rotation rate	= 7.292115×10^{-5} rad/sec
Earth Rotation rate "squared"	= 5.3174943×10^{-9} rad/sec^2
"SE Book" Earth Radius (Re)	= **6378.00 km = 6,378,000 m**
	= 20,925,197 = 2.0925197×10^7 ft
Nom. Total Ribbon Mass	= **865,077 kg**
	= 1,907,170 = 1.907170×10^6 lbm
	= 59,276.8 = 5.92768×10^4 slugs
Ribbon Modulus of Elasticity	= **1.2996×10^{11} N/m^2** (Pascals)
	= 188,500,00 = 1.885×10^8 psi
Gravity Accel at Ballast radius	= **0.03522 m/sec^2**
	= 0.11556 ft/sec^2
	= 0.0036 g's
Centrip. Accel at Ballast radius	= **0.5656 m/sec^2**
	= 1.8558 ft/sec^2
Gravity Force on nom ballast	= **22,357 N**
	= 5,026 lbf
Centrif. Force on nom ballast	= **358,775 N**
	= 80,656 lbf
Net force to "swing" nom ballast	= **336,419 N**
	= 75,630 lbf

"R_{earth}/Ribbon Len" ratio = 0.06378

$\sqrt{Re/R_L}$ = 0.25255

Accel of Grav at GEO approx = **0.3048 m/sec^2**

= 1.0 ft/sec^2

Some related Parameter values as they appear in GTOSS

Accel Grav @earth surface = **9.798222 m/sec^2**

= 32.1464 ft/sec^2

Earth Revolution Period = 86,164.1 sec

Nom. Radius vector to Ballast = **106,377.96 km**

= **10.637796 x10^7 m**

= 349,009,071 = 3.4900971x10^8 ft

Nom. Earth Radius (Re) = **6378.16 km = 6,378,160.1 m**

= 20,925,722 = 2.0925722 x10^7 ft

1.1.2 Ribbon Attribute Constants

<u>All deductions are based on the following assumptions/ribbon properties</u>:

With the exception of the total operational ribbon *<u>elongation</u>* (a value determined via GTOSS), these deductions are based on classical linear string theory and basic physics relationships, and thus only reflect ball-park values and should be applied with temperance.

Young's Modulus (E) = **1,000 GPa** (= 10^{12} N/m^2 = 1.45x10^{12} psi)

Elastic (load bearing) Cross sect. at Ground = **3 mm^2** (= 3.00x10^{-6} m^2 = .00465 in^2)

Elastic (load bearing) Cross sect. at GEO = **7.68 mm^2** (= 7.68x10^{-6} m^2 = .0119 in^2)

Stretched length of tapered Ribbon = **100,000 km** (3.2808x10^8 ft)

Un-Stretched length of tapered Ribbon = **95,292 km** (= 3.1264x10^8 ft)

Total Elongation of tapered Ribbon = **4,708 km** (= 15,446,000 ft = 2,925 miles)

<u>Note</u>: this corresponds to an <u>average</u> <u>strain</u> of 4.9% (data from GTOSS simulation*)*

Ribbon (CNT) <u>intrinsic</u> density = **1.3 g/cm^3** (1.3x10^3 kg/m^3 =2.52 slug/ft^3 = 81.1 lbm/ft^3)

Ribbon <u>lineal</u>-density (@Ground) = **0.0387 g/cm** (3.87 kg/km = 2.60 lbm/1000ft)

Ribbon <u>lineal</u>-density (@GEO) = **0.0997 g/cm** (9.97 kg/km = 6.70 lbm/1000ft)

The term "Lift-off mass", refers to the total mass of the Crawler + Useful cargo. These calculations do not make a distinction between what portion of Lift-off mass is useful, and what is purely "crawler mechanism related". In particular, for practical operations, Lift-off mass may have to be notably less than the static ribbon tension of 20 tons (or 40,000 lbs); See items 3 thru 8 below.

1.1.3 Elevator Dynamical Attribute Values

Based on the above data, we can make the following deductions:

1.1.3.1 Stress wave propagation speed

<u>Transmission Speed of Stress disturbance (longitudinal ribbon wave)</u>

= **27,700 m/s**
= **27.7 km/s**
= 91,000 ft/sec
= 62,000 mph

This parameter depends only upon the material's modulus and intrinsic density; the above values would be very approximate due to the composite nature of the ribbon, and are based on CNT intrinsic properties alone.

<u>Point of Interest</u>: this is the fastest that mechanical information can be transmitted within the CNT ribbon material; for instance, if the ribbon were cut at the ground, it would take on the order of 1 hour for the Ballast mass to get the message, that is, for the zero stress condition at the anchor to propagate up to the ballast. GTOSS has verified this end-to-end transmission time (transmission time is a function of the "number of nodes" and "integration interval" for discrete simulation, approaching the theoretical value as both progress towards "finer" values).

1.1.3.2 Transverse wave propagation speed

<u>Speed of propagation of Transverse ribbon waves</u>

= **6766 m/s**
= **6.77 km/s**
= 22,200 ft/s
= 15,136 mph

This parameter depends upon the tension and lineal-density of the ribbon material.

<u>Point of Interest:</u> If you throw an instantaneous "transverse kink" in the ribbon at the ground, it will take on the order of **45 sec** to reach LEO regions (ie. 300 km altitude), about **1.5 hrs** to reach **GEO**, and about **4 hrs** to reach the **ballast mass** (making a round trip excursion of a transverse wave reflected off the ballast mass be 8 hrs).

<u>Point of Interest:</u> Comparing propagation speeds of Longitudinal versus Transverse ribbon waves, it is seen that the **lowest string-mode frequency** is about **1/4** that of the **lowest longitudinal-mode frequency**.

<u>Point of Interest:</u> Avoiding LEO debris may be practical via SE base-motion. If a re-positioning of the order of a 100-200 meters is sufficient, then this might be attainable via a sliding mechanism on the base station as opposed to moving the anchor platform itself, thus providing quick response. This, combined with the short time to LEO would allow very quick response to a debris detection event. Thus ocean-based stations, and/or over-the-horizon radar debris detection may be a viable scheme.

<u>Point of Interest:</u> Avoiding near GEO debris will require some planning; however, objects in the GEO regime are moving much slower, and spend far more time visible above the radar horizon thus it is less likely to sneak up on the SE as is possible un-charted LEO objects.

1.1.3.3 Effective end-to-end spring rate

<u>Effective End-to-End spring rate (K_{eff}) at the Anchor</u>

= **.039 N/m** (1 N => 26 m)
= **39 N/km**
= .0027 lb/ft (1 lb => 370 ft)

This is based on operational Tension/Total Elongation (from GTOSS data), and also using classical strain-modulus relationships (with average ribbon attributes at Ground and GEO), yielding about the same result. Notes examine implications of K_{eff} .

<u>Point of Interest:</u> Based on a classical uniform string calculation, the value of K_{eff} is bounded below by a calculation corresponding to a "uniform ribbon" based on the elastic attributes at the anchor (yielding a K_{eff} = .030 N/m); the upper bound corresponds to elastic attributes at GEO (yielding a K_{eff} =.047 N/m).

<u>Point of Interest:</u> The above calculation of static K_{eff}, does not mean that such a spring rate would necessarily be realized under significantly Transient conditions, since the classical end-to-end spring rate is based on the entire length of a string instantaneously elongating to participate in resisting applied load. This is because it would take an hour for a strain gradient induced by load (at the earth) to distribute itself along the entire length of the ribbon, thus time become a factor. However, if the system transients were left to damp-out, then the final static elongation would be consistent with this type calculation of K_{eff}.

<u>Point of Interest:</u> Consider an SE with an attached payload whose weight is just equal to the static tension (ie. an essentially neutrally buoyant elevator); now imagine that you could step onto the payload, adding your weight to it, and further suppose that you and the payload were free to translate downward freely (say, over a deep pit in the ground); your added weight would cause the payload to ultimately sink down about 17,000 meters (ie. 55,000 ft or about 10 miles).

1.1.3.4 Climber bobbing period

<u>Liftoff Mass Bobbing Period</u>

Period = 1/Frequency = **1.04 hrs**

This parameter has been simplistically calculated based on a simple 2 DOF dual-mass spring system (assuming the spring is massless) and a lift-off mass of 16,490 kg (18 tons = 36,360 lb_m).

<u>Point of Interest:</u> If one had, say, an **18** ton Liftoff mass attached to a **20** ton-tensioned SE ribbon, and then cut the restraining link between the payload and ground, then a yo-yo bobbing-type oscillation would ensue, with the payload bounding upwards towards the ballast for about 1/2 hr, then bounding back downward, this whole process taking a little over an hour for a classical spring-mass system. Related SE phenomenon could actually take somewhat longer because as the liftoff mass rises, gravity <u>decreases</u> and centripetal acceleration <u>increases</u>, both effects attributing to a greater altitude (amplitude) being realized than predicted by a simple spring mass system.

Point of Interest: Simulations of this using GTOSS indeed exhibits these tendencies (with the above characteristic frequency appearing). In this case, the resultant response was not constrained, resulting in elevator and lift-off mass both moving away from the earth. Thus, other complex non-linear behaviors manifest themselves, such as: motion of ballast and lift-off masses as whole away from and through the earth's inverse square gravity field, excitation of longitudinal modes due to a sharp change in tension as the anchor was severed, finite times of longitudinal wave transmission/and strain-relaxation, etc.

1.1.3.5 Climber bobbing amplitude

Liftoff Mass Bobbing-Mode Amplitude

= **822,000 m**
= 822 km
= 510 miles

This amplitude depends upon the relationship of the static tension to the Lift-off mass. The calculation uses the same data as in 1.1.3.4 above, except, simplistically assumes a single mass-spring system (ie. the ballast mass is assumed "fixed", otherwise the motion is unbounded).

Point of Interest: While this oscillation amplitude would likely never be realized in practice, rather it is included to promote an understanding of the compliant nature of the climber-ribbon system.

1.1.3.6 Climber acceleration potential

Payload Bobbing-Mode Immediate Acceleration Potential

= **1.08 m/sec^2**
= 0.11 g

Point of Interest: This value is arbitrarily based on the free release of an 18 ton Lift-off mass on a ribbon tensioned at 20 tons (the acceleration thus being the ratio of 2 tons/18 tons). Given the low spring rate of the ribbon, a free-release represents the maximum practical acceleration attainable at liftoff.

1.1.3.7 Transverse string mode periods

<u>Classical 1st and 2nd String-Mode Transverse Periods</u> (approx)

1st mode period = **7.5** hrs
2nd mode period = **3.8** hrs

1.1.3.8 Pendulus libration periods

<u>Pendulus In-Plane Period</u> (approx)

= **5.65** days
= 135.6 hrs

<u>Pendulus Out-of-Plane Period</u> (approx)

= **0.98** days
= 23.95 hrs

1.1.3.9 Coriolis effects

Coriolis Acceleration:

The following table quantifies the Coriolis acceleration associated with certain rates of traversal of the elevator Ribbon, and provides typical related force associated with a Climber mass of **16,490 kg (18 tons = 36,360 lbm)**.

Traverse **Rate** (m/s)	Traverse **Rate** (km/hr)	Coriolis **Acc** (m/s^2)	Effec. **Force** (N)	Effec. **Force** (lbf)
50	180	.0073	134	30
100	360	.0146	328	60
200	720	.0292	656	120

<u>Point of Interest:</u> Consider the classical "Clothes-Line" effect (force resolution) in which the <u>tension</u> in a line is employed to equilibrate a <u>normal</u> load by way of <u>deflecting</u> the "clothes-line" (a tangent of small angle effect). If a line is under 178,000 N (40,000 lbf) of tension, then a 0.04 deg deflection would equilibrate a normal force of 267 N (60 lbf); this corresponds to a reverse effective Coriolis force of a 100 m/s traversal rate.

For the earth's rotation rate, the Coriolis relationship used is:

$$\boldsymbol{A}_{cor} = 1.46\boldsymbol{x}10^{-4}\,\dot{\boldsymbol{R}} \text{ (m/s}^2\text{)}$$

1.1.3.10 Ribbon tangential velocity

Tangential Velocity at GEO and Ballast Altitudes:

@ GEO	V_T	= **2,580 m/s**
		= 8,460 ft/s
@ Ballast	V_T	= **7292 m/s**
		= 23, 900 ft/s

Point of Interest: This is tangential velocity relative to the anchor point. Note also that V_T at the Ballast is of the order of the low earth orbit speed.

1.2 ATTRIBUTES OF “CONSTANT STRESS” SE DESIGN

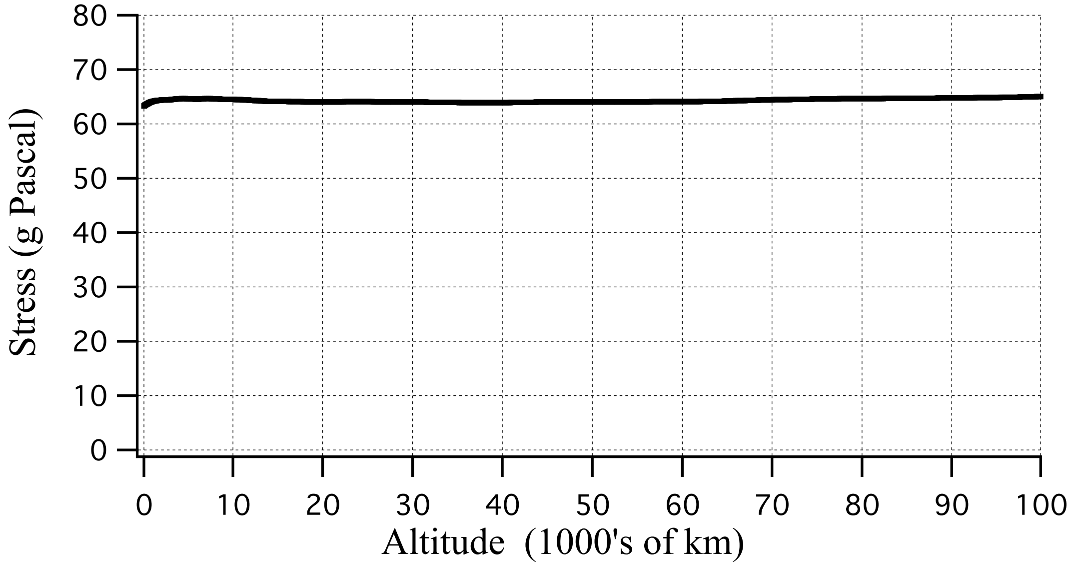
The classic problem of SE design is to achieve the most efficient use of ribbon material; thus, ideally, no section of ribbon is ever under or over-utilized, meaning that nominally, each point on the ribbon experiences the same level of stress, that level being the nominal operating stress with an assumed factor or safety of 2 for example. Typically values of operating stress for the ribbon have been quoted in the 50-60 giga-Pascal range. Thus material.

> **Point of Interest**: Common Scotch tape if it were made of material capable of sustaining 60 GPa stress, could suspend two Cadillac automobiles (11,000 lbs).

The intrinsic technical problem that has historically confronted the SE is been the fact that it must suspend not only the weight of a payload, but more significantly, the weight of itself. Every ribbon particle below GEO altitude creates net force down, and above GEO, a net force upward. GEO then becomes the point at which an SE ribbon would experience the greatest force, and thus have the greatest cross-sectional area; thus proceeding in either direction from GEO, an maximally efficient ribbon would taper ever smaller. The relationship governing the ribbon’s cross sectional area as a function of altitude to achieve this efficiency (uniform stress) is well known, and involves both the material density and strength in exponential form. This leads to the exponentially tapered ribbon design, with maximum width at GEO, tapering to lesser widths at both the ground and the ballast mass. Such design takes into account both inverse square earth gravity as well as linearly altitude-dependent centrifugal force.

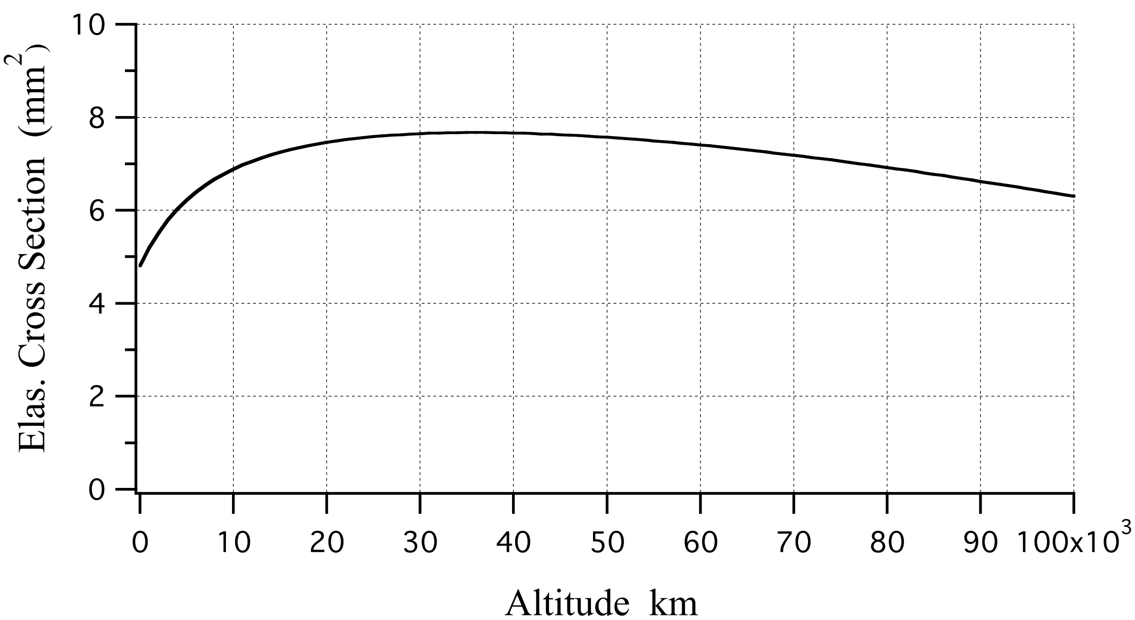
It is this resulting dual tapered ribbon profile that is used within GTOSS to generate the dynamic responses summarized in this handbook. More details on the GTOSS simulation of the tapered ribbon are found in Appendix D

Thus, if one designs the ribbon taper properly to produce uniform material efficiency, and then simulates the SE in an environment properly representing planetary and dynamic effects, then, the resulting stress profile (ie. stress as a function of altitude along the ribbon) should so manifest uniformity. This was done with GTOSS, and the result is shown in the plot below:

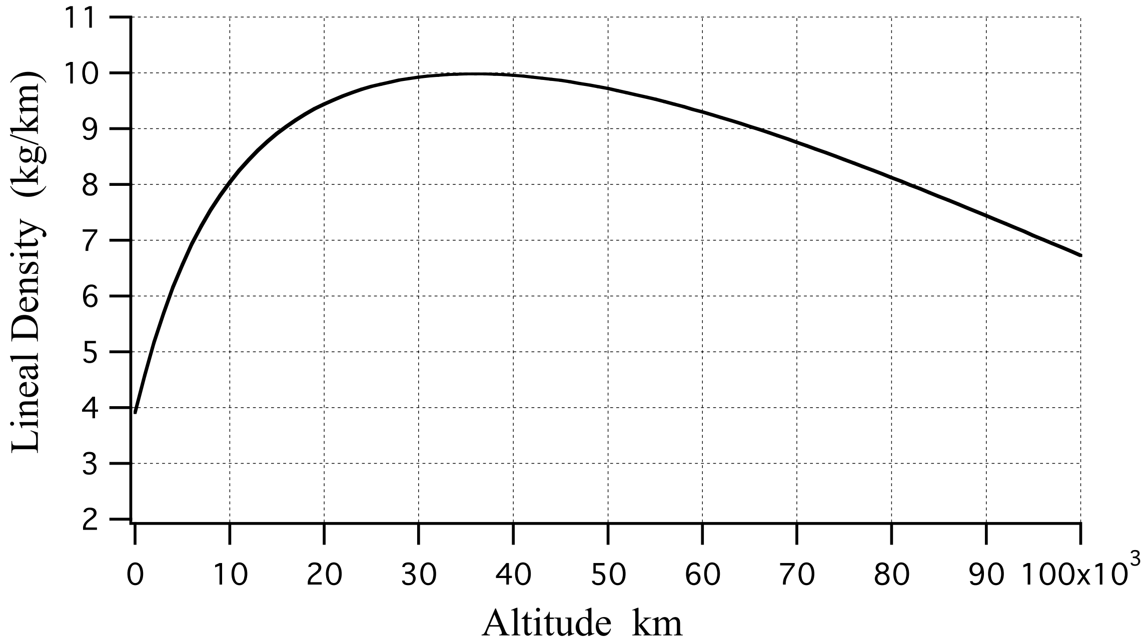


1.2.1 Tapered Ribbon Properties Plots

This uniform stress plot (above) results from the interplay of the ribbon's design profile for density, exponentially tapered elastic cross sectional area, and modulus that was used within GTOSS (shown in the figures below). The slight droop in the stress curve near the earth can be attributed to approximation errors associated with curve-fitting the ribbon profile's taper gradient near the earth.



Based on the elastic cross sectional area profile and a nominal value of ribbon material's bulk density of 1.3 gm/cm^3, the lineal density profile shown below was derived for use in GTOSS.



Lineal Density (kg/km)
11
10
9
8
7
6
5
4
3
2
0
10
20
30
40
50
60
70
80
90
100x10^3
Altitude km

1.2.2 Ribbon Taper Tabulated Properties

All data below is consistent with that provided by Dr. Bradley Edwards, even though the data is presented below at different levels of resolution.

1.2.2.1 High resolution tabular data

High resolution Tabular data: Earth's Surface to GEO altitude ("Radius" is the distance from the center of the earth to a point on the ribbon).

Radius (m)	**Tension** (N/m^2)	**Elastic Area** (m^2)	**Ribbon mass** (kg)
6378000	1.95E+05	3.01E-06	0
7378000	2.29E+05	3.52E-06	3908.9
8378000	2.58E+05	3.97E-06	8484.7
9378000	2.85E+05	4.38E-06	13652.1
10378000	3.08E+05	4.73E-06	19342.6
11378000	3.28E+05	5.05E-06	25496.1
12378000	3.46E+05	5.33E-06	32059.9
13378000	3.63E+05	5.58E-06	38988.6
14378000	3.77E+05	5.80E-06	46242.6
15378000	3.90E+05	6.00E-06	53787.6
16378000	4.02E+05	6.18E-06	61593.4
17378000	4.13E+05	6.35E-06	69633.9
18378000	4.22E+05	6.49E-06	77885.7
19378000	4.31E+05	6.63E-06	86328.3
20378000	4.39E+05	6.75E-06	94943.3
21378000	4.46E+05	6.86E-06	103714.4
22378000	4.52E+05	6.95E-06	112626.9
23378000	4.58E+05	7.04E-06	121667.3
24378000	4.63E+05	7.12E-06	130823.8
25378000	4.68E+05	7.20E-06	140085.3
26378000	4.72E+05	7.26E-06	149441.9
27378000	4.76E+05	7.32E-06	158884.3
28378000	4.79E+05	7.38E-06	168404.1
29378000	4.83E+05	7.42E-06	177993.5
30378000	4.85E+05	7.47E-06	187645.2
31378000	4.88E+05	7.51E-06	197352.6
32378000	4.90E+05	7.54E-06	207109.5
33378000	4.92E+05	7.57E-06	216909.9
34378000	4.94E+05	7.59E-06	226748.4
35378000	4.95E+05	7.62E-06	236619.9
36378000	4.96E+05	7.63E-06	246519.6
37378000	4.97E+05	7.65E-06	256442.9
38378000	4.98E+05	7.66E-06	266385.5
39378000	4.98E+05	7.67E-06	276343.4
40378000	4.99E+05	7.67E-06	286312.7

High Resolution Tabular data: GEO altitude to nominal Ballast altitude.

Radius (m)	**Tension** (N/m^2)	**Elastic Area** (m^2)	**Ribbon mass** (kg)
41378000	4.99E+05	7.68E-06	296289.8
42378000	4.99E+05	7.68E-06	306271.1
43378000	4.99E+05	7.68E-06	316253.4
44378000	4.99E+05	7.67E-06	326233.5
45378000	4.98E+05	7.67E-06	336208.5
46378000	4.98E+05	7.66E-06	346175.3
47378000	4.97E+05	7.65E-06	356131.3
48378000	4.96E+05	7.64E-06	366073.9
49378000	4.95E+05	7.62E-06	376000.4
50378000	4.94E+05	7.61E-06	385908.5
51378000	4.93E+05	7.59E-06	395795.8
52378000	4.92E+05	7.57E-06	405660.1
53378000	4.91E+05	7.55E-06	415499.2
54378000	4.89E+05	7.53E-06	425311.1
55378000	4.88E+05	7.50E-06	435093.7
56378000	4.86E+05	7.48E-06	444845.1
57378000	4.84E+05	7.45E-06	454563.5
58378000	4.82E+05	7.42E-06	464247.2
59378000	4.80E+05	7.39E-06	473894.2
60378000	4.78E+05	7.36E-06	483503.1
61378000	4.76E+05	7.33E-06	493072.2
62378000	4.74E+05	7.30E-06	502599.9
63378000	4.72E+05	7.26E-06	512084.8
64378000	4.70E+05	7.23E-06	521525.3
65378000	4.67E+05	7.19E-06	530920.3
66378000	4.65E+05	7.15E-06	540268.2
67378000	4.63E+05	7.12E-06	549567.8
68378000	4.60E+05	7.08E-06	558817.8
69378000	4.57E+05	7.04E-06	568017.1
70378000	4.55E+05	7.00E-06	577164.5
71378000	4.52E+05	6.95E-06	586258.8
72378000	4.49E+05	6.91E-06	595299.1
73378000	4.46E+05	6.87E-06	604284.2
74378000	4.44E+05	6.82E-06	613213.1
75378000	4.41E+05	6.78E-06	622085.0
76378000	4.38E+05	6.73E-06	630898.8
77378000	4.35E+05	6.69E-06	639653.8
78378000	4.32E+05	6.64E-06	648348.9
79378000	4.29E+05	6.59E-06	656983.5
80378000	4.26E+05	6.55E-06	665556.7
81378000	4.22E+05	6.50E-06	674067.9
82378000	4.19E+05	6.45E-06	682516.2
83378000	4.16E+05	6.40E-06	690900.9
84378000	4.13E+05	6.35E-06	699221.6
85378000	4.10E+05	6.30E-06	707477.4

High Resolution Tabular data: GEO altitude to Nominal Ballast altitude (continued)

Radius (m)	**Tension** (N/m^2)	**Elastic Area** (m^2)	**Ribbon mass** (kg)
86378000	4.06E+05	6.25E-06	715667.8
87378000	4.03E+05	6.20E-06	723792.3
88378000	4.00E+05	6.15E-06	731850.4
89378000	3.96E+05	6.10E-06	739841.4
90378000	3.93E+05	6.04E-06	747765.0
91378000	3.89E+05	5.99E-06	755620.7
92378000	3.86E+05	5.94E-06	763408.0
93378000	3.82E+05	5.88E-06	771126.6
94378000	3.79E+05	5.83E-06	778776.2
95378000	3.76E+05	5.78E-06	786356.2
96378000	3.72E+05	5.72E-06	793866.5
97378000	3.68E+05	5.67E-06	801306.7
98378000	3.65E+05	5.61E-06	808676.6
99378000	3.61E+05	5.56E-06	815976.0

High Resolution Tabular data: Nominal Ballast altitude to Beyond.

Radius (m)	**Tension** (N/m^2)	**Elastic Area** (m^2)	**Ribbon mass** (kg)
100378000	3.58E+05	5.51E-06	823204.5
101378000	3.54E+05	5.45E-06	830310.5
102378000	3.51E+05	5.40E-06	837448.4
103378000	3.47E+05	5.34E-06	844463.4
104378000	3.44E+05	5.29E-06	851406.9
105378000	3.40E+05	5.23E-06	858278.9
106378000	3.36E+05	5.18E-06	865079.2
107378000	3.33E+05	5.12E-06	871807.8
108378000	3.29E+05	5.07E-06	878464.6
109378000	3.26E+05	5.01E-06	885049.6
110378000	3.22E+05	4.95E-06	891562.8
111378000	3.18E+05	4.90E-06	898004.2
112378000	3.15E+05	4.84E-06	904373.8
113378000	3.11E+05	4.79E-06	910671.6
114378000	3.08E+05	4.73E-06	916897.8
115378000	3.04E+05	4.68E-06	923052.5
116378000	3.01E+05	4.62E-06	929135.6
117378000	2.97E+05	4.57E-06	935147.4
118378000	2.93E+05	4.52E-06	941088.0
119378000	2.90E+05	4.46E-06	946957.6
120378000	2.86E+05	4.41E-06	952756.2
121378000	2.83E+05	4.35E-06	958484.1

High Resolution Tabular data: Nominal Ballast altitude to Beyond (continued)

Radius (m)	**Tension** (N/m^2)	**Elastic Area** (m^2)	**Ribbon mass** (kg)
122378000	2.79E+05	4.30E-06	964141.6
123378000	2.76E+05	4.24E-06	969728.7
124378000	2.72E+05	4.19E-06	975245.8
125378000	2.69E+05	4.14E-06	980693.0
126378000	2.65E+05	4.08E-06	986070.7
127378000	2.62E+05	4.03E-06	991379.2
128378000	2.59E+05	3.98E-06	996618.6
129378000	2.55E+05	3.92E-06	1001789.3
130378000	2.52E+05	3.87E-06	1006891.6
131378000	2.48E+05	3.82E-06	1011925.9
132378000	2.45E+05	3.77E-06	1016892.4
133378000	2.42E+05	3.72E-06	1021791.6
134378000	2.38E+05	3.67E-06	1026623.8
135378000	2.35E+05	3.61E-06	1031389.3
136378000	2.32E+05	3.56E-06	1036088.5
137378000	2.28E+05	3.51E-06	1040721.8
138378000	2.25E+05	3.46E-06	1045289.7
139378000	2.22E+05	3.41E-06	1049792.5
140378000	2.19E+05	3.36E-06	1054230.7
141378000	2.16E+05	3.32E-06	1058604.7
142378000	2.12E+05	3.27E-06	1062914.9
143378000	2.09E+05	3.22E-06	1067161.7
144378000	2.06E+05	3.17E-06	1071345.7
145378000	2.03E+05	3.12E-06	1075467.3
146378000	2.00E+05	3.08E-06	1079526.9
147378000	1.97E+05	3.03E-06	1083525.1
148378000	1.94E+05	2.98E-06	1087462.3
149378000	1.91E+05	2.94E-06	1091339.1
150378000	1.88E+05	2.89E-06	1095155.8

1.2.2.2 Low resolution tabular data

This data was derived from GTOSS table input, and represents a minimum-data optimal fit of the ribbon taper (assuming a quadratic interpolation for intermediate points). The format of this data is characterized by Region definitions; thus a Region is defined by the Region-Length (as opposed to radius from earth center), and there is then a bounding value of a parameter at each end of the Region. Note that the sum of the Region lengths nominally would total to the ballast altitude.

Low resolution Tabular data:

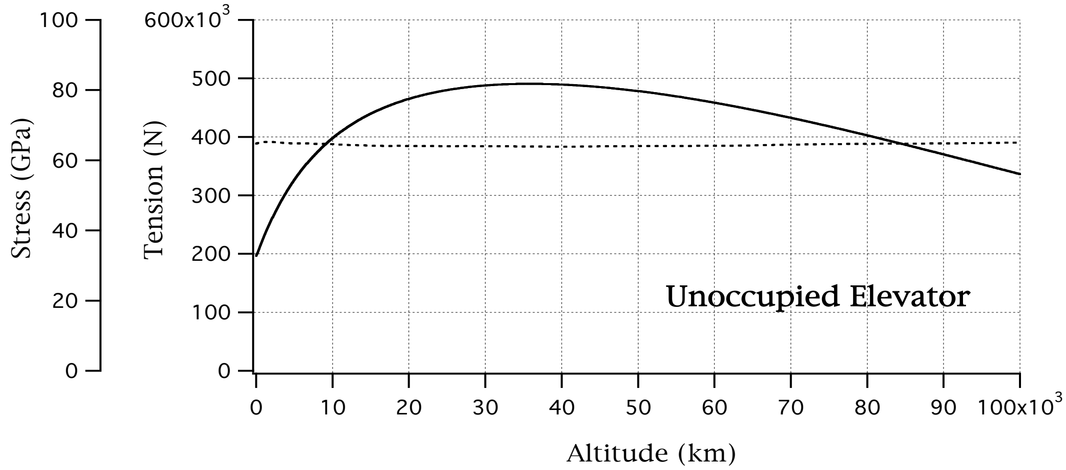
Earth's surface to nominal Ballast altitude by Regions

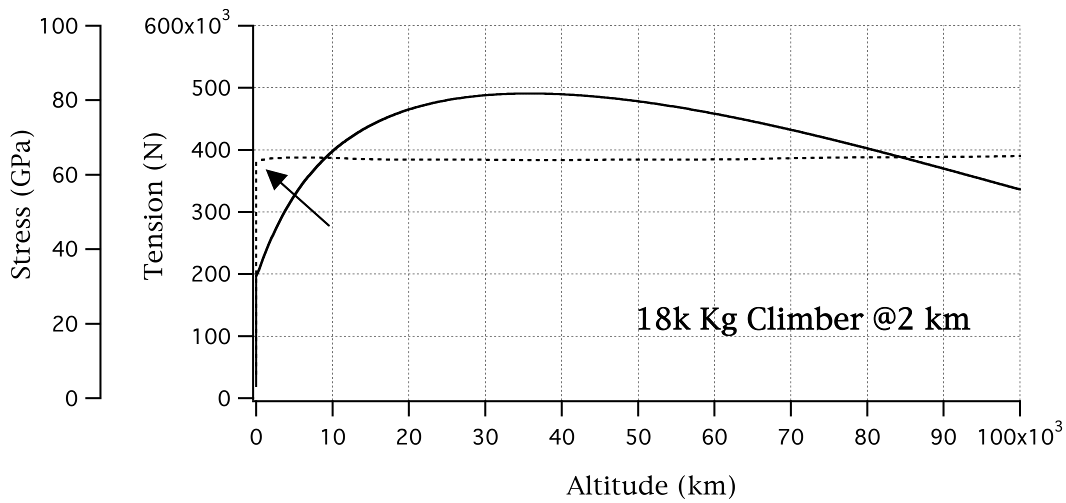
Note: this represents an optimal approximation to the ribbon length-varying parameters, assuming quadratic interpolation is used evaluate properties intermediate to the region boundaries.

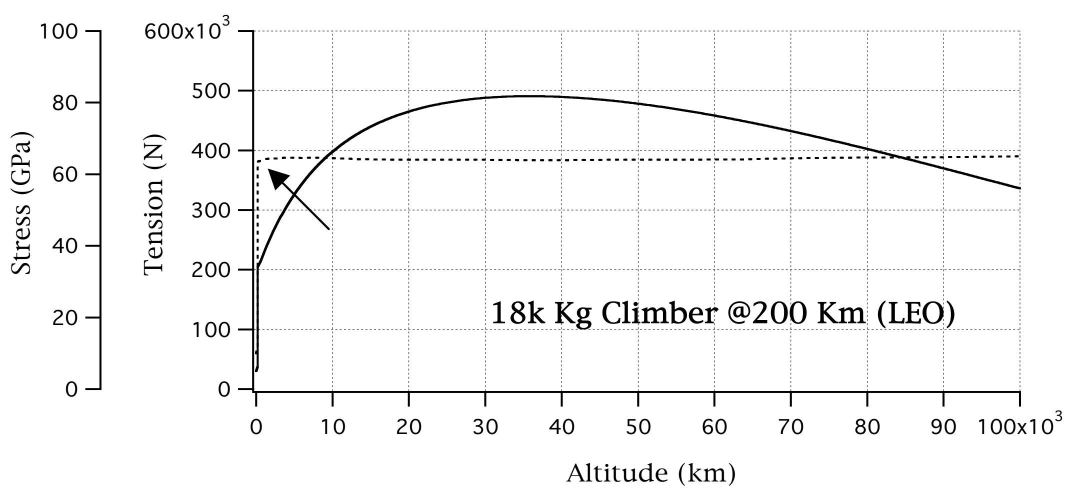
Lineal Density (kg/km)	**Cross Section** (mm^2)	**Region Δ Length** ΔL (km)
3.9089	3.010	<= value at Earth
		3048
5.6550	4.3825	
		3048
6.8902	5.3298	
		3048
7.8054	5.9981	
		4572
8.7057	6.7012	
		4572
9.2712	7.1393	
		3048
9.5242	7.3331	
		6096
9.8442	7.5666	
		6096
9.9707	7.6659	
		6096
9.9558	7.6659	
		6096
9.8368	7.5666	
		9144
9.5242	7.3331	
		9144
9.0778	6.9957	
		15240
8.1849	6.2656	
		12192
7.3366	5.6398	
		8656.32
6.7265	5.1689	<= value at Ballast

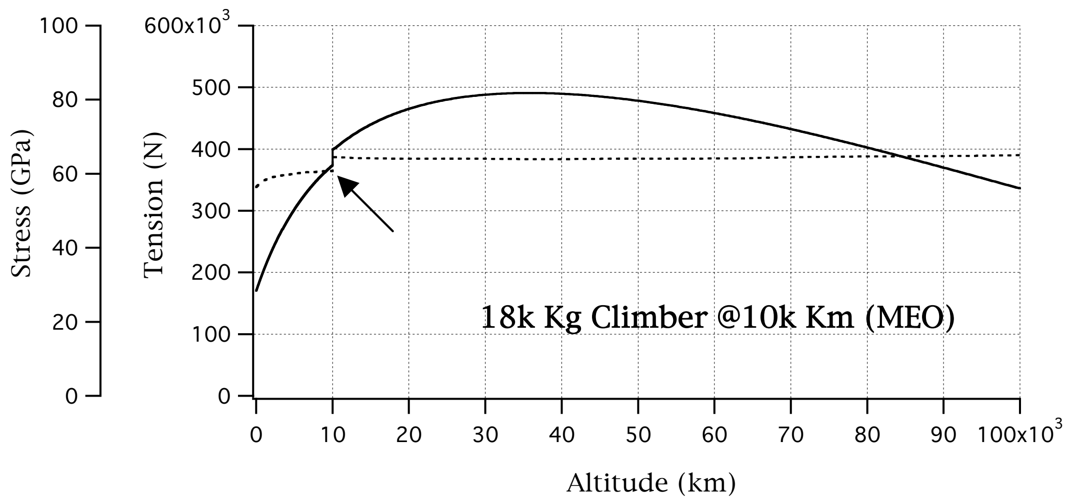
1.3 NOMINAL TENSION AND STRESS STATES

The graphs below illustrate the tension and stress profiles associated static positioning of a 18,000 kg climber at various altitudes. From this, similar tension profiles for an object at other altitudes become readily apparent.

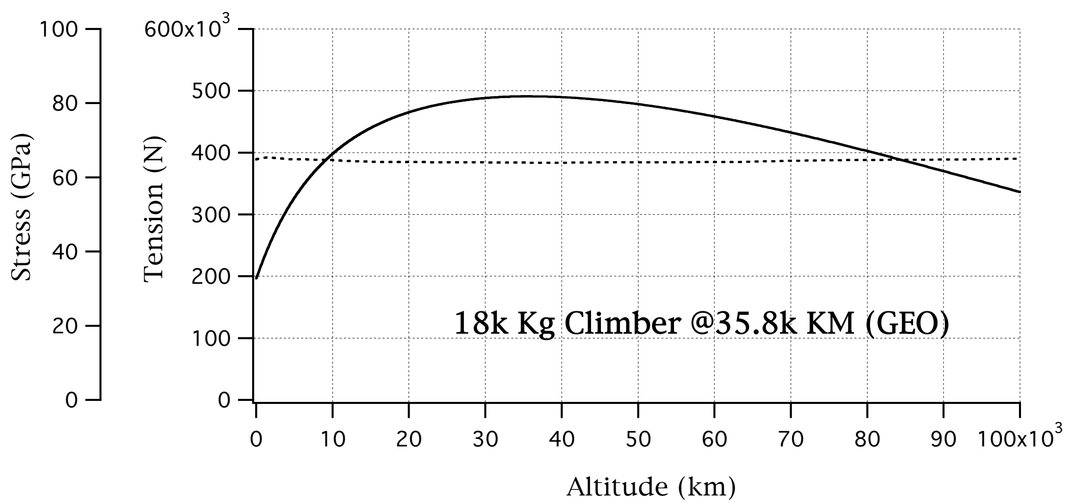




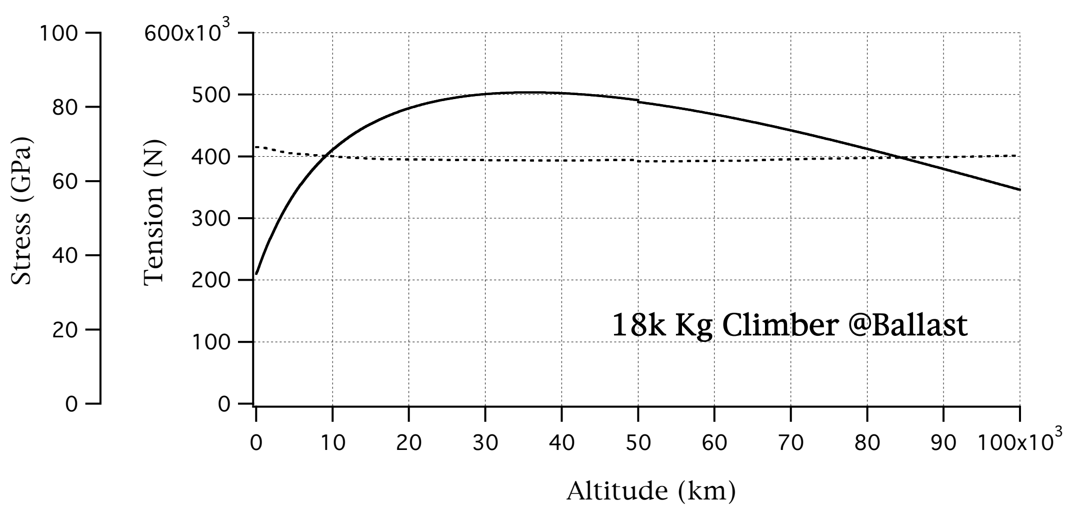




Stress (GPa)
Tension (N)
600x10^3
18k Kg Climber @10k Km (MEO)
Altitude (km)
100x10^3

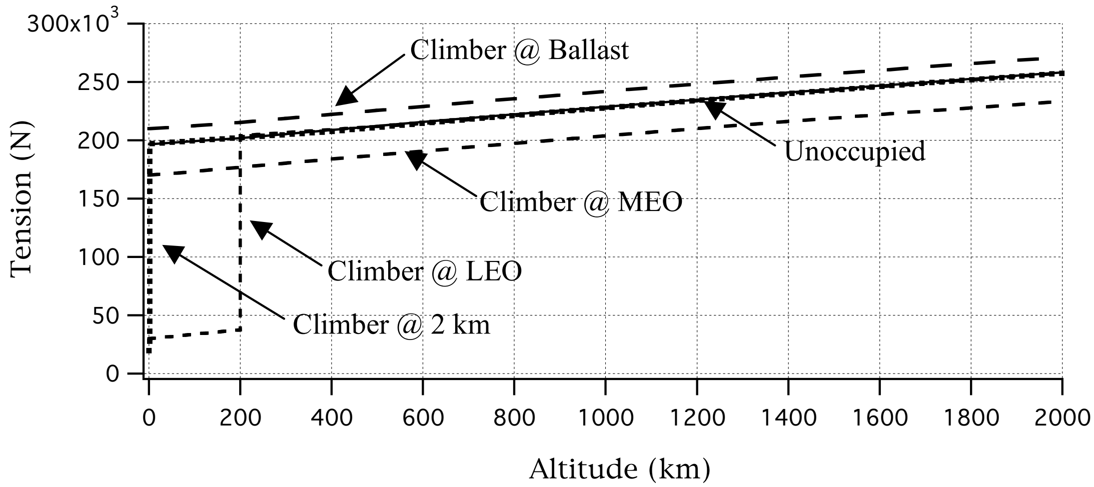


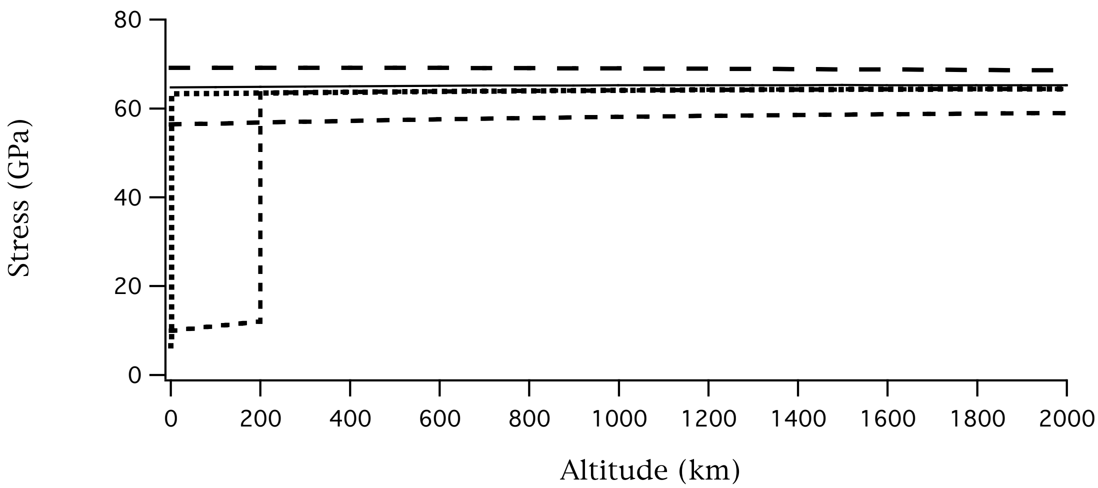
Stress (GPa)
Tension (N)
600x10^3
18k Kg Climber @35.8k KM (GEO)
Altitude (km)
100x10^3



Stress (GPa)
Tension (N)
600x10^3
18k Kg Climber @Ballast
Altitude (km)
100x10^3

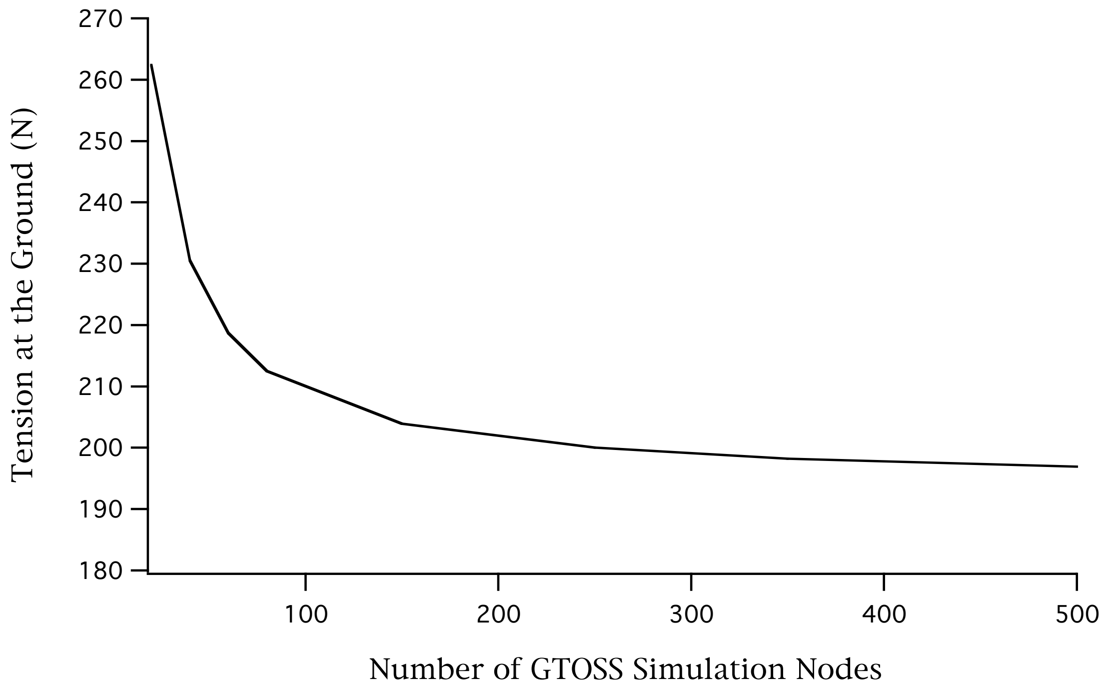
The two graphs below provide a low altitude magnified composite view of tension and stress profiles for all the above cases.





1.4 GRAVITY-WELL SIMULATION CONVERGENCE

To properly simulate a space elevator configuration using a discrete nodal approach, one must pay attention to the spatial-convergence of the simulation as it pertains to the "gravity well" (ie. inverse-square planetary gravity model). The graph below depicts (for the Earth) the tension at the anchor point as a function of the number of "uniformly-spaced" nodes used to simulate the elevator ribbon.



It can be seen that for node-counts greater than 200, the earth's gravity-well is being fairly well acknowledged. For lesser node counts, simulation results may still be acceptable, depending upon the degree to which total-available anchor tension must be fully realized.

Note also, that *non-uniform* distributions of lesser node-counts (for instance 100 nodes from ground to 10,000 km, then 50 nodes between there and ballast, etc) may suffice to fully acknowledge the gravity well.

1.5 USEFUL FORMULAS FOR TETHER DYNAMICS

1.5.1 Definition of Symbols

σ = Stress

ε = Strain

T = Tension

L = Length

E = Young's modulus

d = Elastic diameter

A = Elastic cross sectional area

ρ = Volumetric mass density

ρ_L = Lineal mass density (= $\frac{\rho}{A}$)

β = Material intrinsic damping factor (sec)

δ = End-to-End tether elongation

K_{eff} = Effective end-to-end spring constant

K_{Deff} = Effective end-to-end damping constant

1.5.2 From Classical Small Amplitude String Theory

Transverse: Natural Frequencies $f^T{}_n$

$$f_n^T = \frac{n}{2L}\sqrt{\frac{T}{\rho_L}}$$

$$= \frac{n}{2L} S^T$$

Wave Propagation Speed S^T

$$S^T = \sqrt{\frac{T}{\rho_L}}$$

Longitudinal: Natural Frequencies $f^L{}_n$

$$f_n^L = \frac{n}{2L}\sqrt{\frac{AE}{\rho_L}}$$

$$= \frac{n}{2L} S^L$$

Stress Propagation Speed S^L

$$S^L = \sqrt{\frac{E}{\rho}}$$

$$= \sqrt{\frac{AE}{\rho_L}}$$

1.5.3 Plumb-Bob (Yo-Yo) Oscillations

Plumb Bob (Longitudinal) Frequency (2 masses and a spring):

$$f_n^{YoYo} = \frac{1}{2\pi}\sqrt{\frac{K_{eff}\,(M_1 + M_2)}{M_1\,M_2}}$$

1.5.4 Elastic Diameter -vs- End-to-End Spring Rate

Relationship of Equivalent Elastic Diameter to the tether Effective End-to-End Spring-Rate Constant:

$$d = \sqrt{\frac{4\,L\,K_{eff}}{\pi\,E}}$$

$$K_{eff} = \frac{A\,E}{L}$$

1.5.5 Damping Relationships

Definition of **Equivalent End-to-End Damping Constant** K_{Deff}: $T_{Dexp} = K_{Deff}\,\frac{d\delta}{dt}$

Where: T_{Dexp} is the tension experienced at attach points *due to damping alone*.

Definition of **Material Intrinsic Damping Factor** β: $\sigma = E(\varepsilon + \beta\,\frac{d\varepsilon}{dt})$

Relationship between Equivalent End-to-End Damping constant and the conventional Spring-Rate constant.

$$K_{Deff} = \beta\,K_{eff}$$

1.5.6 Segment -vs- Overall Effective-Attributes Relations

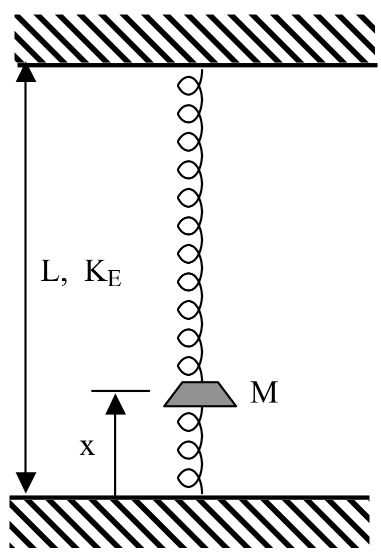
Relationship between Equivalent End-to-End constants, and an N segment discrete (bead modeled) tether:

$K_{seg} = N K_{eff}$ Where: K_{seg} = Seg-equivalent to the End-to-end spring constant

$K_{D_{seg}} = N K_{D_{eff}}$ Where: $K_{D_{seg}}$ = Seg-equivalent to the End-to-end damping constant

1.5.7 Stretched Uniform Spring with Intermediate Mass

This addresses the effect upon oscillation frequency of positioning a concentrated mass, **M**, at an intermediate distance **x** from one end, along a stretched uniform spring of length **L** and an End-to-End Effective spring rate of $\mathbf{K_E}$. The applicability of this simplified model is to approximate Longitudinal Bobbing-Frequencies of a climber mass at any point on the elevator ribbon from ground to ballast.



The circular frequency ω is:

$$\omega = \omega_E \sqrt{\frac{1}{\lambda(1-\lambda)}} \qquad \text{with: } \boldsymbol{f} = \frac{\omega}{2\pi} \qquad \text{and Period, } \boldsymbol{T} = \frac{1}{\boldsymbol{f}}$$

$$\text{where: } \lambda = \frac{\boldsymbol{x}}{\boldsymbol{L}} \qquad \text{and, } \quad \omega_E = \sqrt{\frac{\boldsymbol{K}_E}{\boldsymbol{M}}}$$

Note that ω_E is simply the circular frequency of mass M, under the action of a single spring with spring constant $\boldsymbol{K}_E$ (which is the lowest effective spring rate associated with the ribbon). The frequency ω_E would be that of the climber positioned near the anchor (or ballast), while *assuming* the (short) ribbon between the climber and anchor (or ballast) does not exist; ω_E would be the *lowest* longitudinal Bobbing frequency associated with the Climber and the elevator-ribbon-system; the longitudinal Bobbing frequency of the ballast being the lowest). Note that the frequency ω_E is NOT close to values of ω corresponding to *small values of* λ, as these frequencies could be quite high (due to the high spring rate of the short ribbon segment between the climber and either end). Values of ω corresponding to small values of λ may only manifest in piecewise fashion due to potential discontinuous non-linear slack-taut motion that may ensue with an actively bobbing climber near either end of the ribbon.

Below is shown a typical plot of Longitudinal Bobbing Period corresponding to values of λ spanning climber positions from the ground to the ballast for the nominal elevator configuration addressed in this handbook.