## Space Elevator

# Dynamics Reference Manual 

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Contains Sections 3 and 4 Only

## TABLE OF CONTENTS

3.0 DEBRIS AVOIDANCE ..... 5
3.1 DEBRIS ENVIRONMENT ..... 5
3.2 DEBRIS-AVOIDANCE TIMING ENVELOPES/RESPONSE ..... 5
3.3 SEA PLATFORM PERFORMANCE DATA ..... 5
3.4 TRANSVERSE WAVES WITH CLIMBER ON RIBBON ..... 5
3.5 TRANSVERSE WAVE DAMPING USING BASE MOTION ..... 5
4.0 AERODYNAMIC RESPONSE ..... 6
4.1 ATMOSPHERIC CHARACTERIZATION ..... 6
4.2 OVERVIEW OF AERODYNAMIC RESPONSE ..... 7
4.3 DETAILED RESPONSE TO A REFERENCE WIND ..... 10
4.3.1 A Typical Aerodynamic Load Distribution Response ..... 10
4.3.2 Extreme Aerodynamic Loading ..... 13
4.3.3 Typical Ribbon Deflection Response Shapes ..... 17
4.3.3.1 Deflections for un-occupied elevator. ..... 17
4.3.3.2 Deflections with climber in low atmosphere ..... 18
4.3.3.3 Deflections with climber at LEO ..... 20
4.4 RIBBON HORIZONTAL DISPLACEMENT ..... 23
4.4.1 Horizontal Displacement vs Wind Speed (Unoccupied) ..... 23
4.4.2 Horizontal Displacement vs Wind Duration (Unoccupied) ..... 24
4.4.3 Horizontal Displacement vs Ribbon Width (Unoccupied) ..... 25
4.4.4 Horizontal Displacement vs Wind Speed (Climber in Atmos) ..... 25
4.4.5 Horizontal Displacement vs Wind Speed (Climber at LEO) ..... 26
4.5 RIBBON DEPARTURE ANGLES ..... 27
4.5.1 Departure Angle vs Wind Speed (Unoccupied) ..... 28
4.5.2 Departure Angle vs Ribbon Width (Unoccupied) ..... 28
4.5.3 Departure Angle vs Wind Speed (Climber in Atmosphere) ..... 29
4.5.4 Departure Angle vs Wind Speed (Climber at LEO) ..... 29
4.6 STRESS RELATED TO WIND RESPONSE ..... 30
4.6.1 Stress Profile vs Wind (Unoccupied) ..... 31
4.6.2 Stress Profile vs Ribbon Width (Cat 0, Unoccupied) ..... 32
4.6.3 Stress Profile vs Wind (Climber in Atmosphere) ..... 33
4.6.4 Stress Profile vs Wind (Climber at LEO) ..... 35

## INTRODUCTION

This manual is a preliminary work aimed at providing a source of convenient dynamicsrelated information for those involved with all aspects of space elevator (SE) development and design. For some, this manual may serve as a primer of SE dynamics, for all, it is a source of specific constants, attributes and SE behaviors. This manual is a work in progress as much remains to be addressed as work proceeds on the project. Each section of the manual addresses a special aspect of information pertaining to the SE.

The dynamics attributes have been mostly derived from the time-domain simulation called GTOSS (Generalized Tethered Object Simulation). An outline of GTOSS is included (in Appendices A through D, etc) to allow the user to assess the pertinence of this simulation in providing such results for each aspect of SE dynamics. Related materials s re-organized and derived from these studies by the author appear in the papers listed among the references.

General Note: Some items below should be included for future efforts
a. Thermal response
b. Climber attitude dynamics
c. Ocean wave effects on longitudinal dynamics
d. Sun-Moon tidal effects
e. Aerodynamic pull-down response
f. Breakage debris-footprints
g. General pull-down response

### 3.0 DEBRIS AVOIDANCE

...THE DEBRIS SECTION IS NOT INCLUDED IN THIS RELEASE...

### 3.1 DEBRIS ENVIRONMENT

- Tracked debris histograms vs altitude bins
- Statistical characterization of untracked (small) debris
- Characterization of ribbon damage vs type of debris


### 3.2 DEBRIS-AVOIDANCE TIMING ENVELOPES/RESPONSE

- Characterize "tracked-debris lead times"
- Characterize "wave transmission time"-vs- debris altitude
- Characterize "transverse wave clearance"-vs- altitude


### 3.3 SEA PLATFORM PERFORMANCE DATA

- Plot of minimum time to transverse displacement (no arrest)
- Plot of minimum time to "arrested" transverse displacement
- Timing considerations for wave cancellation


### 3.4 TRANSVERSE WAVES WITH CLIMBER ON RIBBON

- Plot of transverse wave reflection off of Climber @ LEO
- Plot of transverse wave reflections off of Climber @ MEO
- Plot of transverse wave reflections off of Climber @ GEO
- Plot of transverse wave through-put with Climber @ Launch
- Plot of transverse wave through-put with Climber @ LEO
- Plot of transverse wave through-put with Climber @ MEO
- Plot of transverse wave through-put with Climber @ GEO


### 3.5 TRANSVERSE WAVE DAMPING USING BASE MOTION <br> - Strobe plots of wave cancellation <br> - Timing data for wave cancellation

### 4.0 AERODYNAMIC RESPONSE

This section does not attempt to establish design criteria or limits based on aerodynamic response; rather the intention here is to familiarize the user with inherent mechanisms underlying aerodynamic response, and identify the attributes of response that may be critical in SE design. Much in SE aerodynamic response runs counter to initial intuitive assessments.

Air loads on the SE and the resulting dynamic response were evaluated using the GTOSS subsonic aerodynamic regime capability. More information about the GTOSS air loads model can be found in Appendix B.

### 4.1 ATMOSPHERIC CHARACTERIZATION

The area of the Pacific ocean, considered to be optimal for location for the space elevator, seems to have little quantified data for the wind environment "at altitude" above sea level; thus it was concluded that at present, "probability of occurrence" type of synthesized wind-altitude envelops would likely not be meaningful. Thus this handbook attempts to address the general attributes of SE aerodynamic response to a simplified wind environment for use in overall preliminary assessment of real wind effects as described below.

Thus, for purposes of preliminary assessment, a constant wind-versus-altitude profile was adopted as a reference. The wind level was allowed to buildup linearly with time, starting from no-wind and progressing to full-wind in a period of two hours. This was followed by a period of constant wind at peak level (typically two hours). Following this constant wind period, the wind level decreased linearly with time to zero over a period of two more hours. The figure below, depicts this for the case of a Category 0 Typhoon, termed a "tropical disturbance".


Thus, all wind scenarios started with zero wind, and for those cases of 2 hour peak-wind duration, returned back to zero wind by $21,600 \mathrm{sec}$ (about 6 hrs lapsed time), with runs terminated after 10 hours ( $35,000 \mathrm{sec}$ ) of simulated time. For simplicity of results correlation, all winds blew from West to East, with no northerly component.

This wind categories used were:
(a). Category 0 (average of $25 \mathrm{~m} / \mathrm{s}=55 \mathrm{mph}=81 \mathrm{ft} / \mathrm{s}$ )
(b). Category 1 (average of $33 \mathrm{~m} / \mathrm{s}=74 \mathrm{mph}=107 \mathrm{ft} / \mathrm{s}$ )
(c). Category 2 (average of $40 \mathrm{~m} / \mathrm{s}=90 \mathrm{mph}=132 \mathrm{ft} / \mathrm{s}$ )
(d). Category 3 (average of $54 \mathrm{~m} / \mathrm{s}=120 \mathrm{mph}=176 \mathrm{ft} / \mathrm{s}$ )

## Important notes concerning snapshot plots depicting aerodynamic response:

- Many of the figures in this handbook depict a series of snapshots of the data, taken at discrete times (frequently a series of more or less uniform time intervals of about 2000 sec ).
- For snapshots taken during the initial 2 hour build up of wind, data is depicted by the thinnest solid lines.
- For snapshots taken during the duration of peak wind ( 2 or 4 hours), data is depicted by thicker solid lines.
- For snapshots taken during the 2 hour duration of diminishing wind, data is depicted with long dashes.
- For snapshots taken after the wind has diminished again to zero, data is depicted with finer dashes.


### 4.2 OVERVIEW OF AERODYNAMIC RESPONSE

SE aerodynamic response is characterized by some rather startling and unintuitive attributes. These are:

1. Absence of critical over-stress resulting from ribbon deflection.
2. Apparent ease with which wind load translates ribbon horizontally.
3. Existence of a critical load level, ushering in non-linear response to load.
4. Potential for near-horizontal ribbon departure angles.
5. Sensitivity to wind duration (for winds greater than critical level).
6. Accentuation of all load responses when the elevator is occupied by a climber.

The above synopses are further addressed below:

1. Absence of critical over-stress resulting from ribbon deflection: The absence of overstress under aerodynamic loading is attributable to the following.

- The overall ribbon has an extremely low effective end-to-end spring rate at earth on the order of $.04 \mathrm{~N} / \mathrm{m}$. This mean that relative to the scale of local atmospheric disturbance,
the ribbon can tolerate a significant amount of elongation without significant rise in tension.
- A ribbon departure of even 200 km downrange, while appearing significant from an anchor-station viewpoint, and presenting a bizarre ribbon departure of near horizontal, in fact represents a minimal increase in overall strain for a $100,000 \mathrm{~km}$ long ribbon. Distributed over the length of the ribbon, this corresponds to an increase in strain of about $0.2 \%$ strain, thus insignificant stress increases (note, the SE ribbon nominally operates at about $4 \%-5 \%$ average strain).
- The fact that stress wave propagation time is very short (approximately 1 hour to travel the full $100,000 \mathrm{~km}$ length of the ribbon) compared to the time it takes a strong wind to build up, effectively defuses the possibility of localized stress at the source of the disturbance by quickly propagating stress gradients upward along the entire length of the ribbon, distributing strain.

The SE ribbon's low effective spring rate can be intuitively grasped by imagining a length of ribbon that is, say, 1 km long; among others; one measure of this length's elasticity is the effective end-to-end spring rate (see tether formulas in section 1.4). Now, suppose two such lengths of ribbon were connected end-to-end ("springs in series"); this new, 2 km long piece would exhibit a spring rate of $1 / 2$ the value of either length individually. In general, if N of these spring lengths, each of spring rate K , are connected in series, the resultant end-to-end spring rate of the composite spring is $\mathrm{K} / \mathrm{N}$; thus, the longer the spring (of a given material), the lower the (overall) spring rate. Thus, it becomes evident that the SE ribbon could exhibit an extremely low spring rate. Note that the spring rate is independent of the amount of tension in a spring; that is, the incremental change in tension resulting from an incremental change in length does not depend upon the spring's preload.
2. Apparent ease with which wind load translates ribbon horizontally: The propensity for the wind to blow the ribbon downrange can be attributed to the aerodynamic model and the ribbon geometry as it yields to the relative wind. In order for the ribbon to sustain horizontal displacement, it is necessary for the vertical and horizontal components of air load to equilibrate respectively the appropriate percentage of vertical and horizontal components of ribbon tension. The aerodynamic source for this equilibration arises over a region of essentially uniform curvature as the ribbon departs the horizontal and proceeds upward to vertical. The aerodynamics model used in this study predicts that if the relative wind has any component normal to the ribbon, then a pressure against the ribbon results, and a force normal to a tangent to the ribbon results. The vector integral of this force distribution provides the required horizontal and vertical force components.
3. Existence of a critical load level, ushering in non-linear response to load: As aerodynamic load increases, both vertical as well as horizontal components of air load are created on the ribbon due to the characteristic curvature induced in the ribbon by horizontal deflection under air load. For quasi-steady response, net vertical and horizontal load must be zero. As curvature (deflection) occurs, both horizontal and
vertical components of air load build against the equilibrating tension components. Note that as curvature builds, the tension in the ribbon is essentially unaffected (from item 1 above). At some point, assuming air load does not increase beyond a certain level, due to the geometry of curvature, the horizontal tension component equilibrates the horizontal air load and horizontal deflection stops increasing. As seen from results, when the critical air load level is reached, then the nature of the aerodynamic response changes drastically from a displacement-sensitive equilibrium restoring process to an unconstrained deflection process in which non-linear effects will dictate if and/or when horizontal deflection will become limited. This point is reached when almost simultaneously both the total horizontal and vertical components of air load becoming equivalent to the tension. At this point, the air load essentially replaces the anchor as the source of vertical equilibrium force, and the horizontal component of tension becomes no longer capable of constraining horizontal deflection.
4. Potential for near-horizontal ribbon departure angles: After the critical air load is reached, then the ultimate amount of horizontal deflection becomes unlimited (in a linear sense), and the resulting extreme curvatures of the ribbon will produce departure angles from the anchor of near (if not actually) horizontal as the ribbon progresses down-range in response to air loads.
5. Sensitivity to wind duration (for winds greater than critical level): After the critical air load is reached, then the ultimate amount of horizontal deflection becomes a timedependent phenomenon, depending upon duration of wind above the critical level. This in a sense then dictates that the maximum down-range deflection becomes somewhat dependent upon the geographical extent of the wind field; this being because deflection will simply increase until the wind subside, either due to temporal or geographical extent of the wind field.
6. Accentuation of all load responses when the elevator is occupied by a climber: The above explanations of the extent to which the ribbon can be blown horizontal by the wind alludes to the criticality of that condition in which the wind overpowers the balance between the only restoring mechanism at the ribbon's disposal, namely its tension, and the air loads. Now, any elevator configuration that includes a climber on the ribbon below GEO will reduce the tension in the atmosphere. The lower the climber, the greater is this reduction. The lower the tension in the ribbon, the more easily the wind can overpower the balance between tension and air load. Thus is all cases of a climber on the ribbon, certain aspects of aerodynamic response can be exacerbated; this effect is progressive until the climber is fairly low in the atmosphere; at low altitudes, as the section of ribbon between the climber and anchor point becomes ever shorter, the effect of lowering tension between climber and ground starts to transform the elevator effectively into the unoccupied elevator case. Imagine a case in which the climber is a 50 m altitude; here, it is clear that if one were to stand back and observe the response, the "big picture" would not differ significantly from the unoccupied case.

Section 4.3 below demonstrates the above.

### 4.3 DETAILED RESPONSE TO A REFERENCE WIND

The data shown below represent responses to the type of wind profiles described above.

### 4.3.1 A Typical Aerodynamic Load Distribution Response

The following sets of data correspond to a Category 0 wind and a ribbon width of 10 cm exposed to the lower atmosphere. This case is chosen for in depth description since it exposes many of the unintuitive aspects of SE aerodynamics. In particular the effect of reaching a critical air load level, minimal overstress in the presence of significant horizontal ribbon deflection under air load. Note that the air loads have only "doubled" over the case of the 5 cm ribbon.

Shown below are deflection snapshots over a period of 6 hours. Superimposed on this graph is the maximum deflection for the same wind, but with a ribbon width of 5 cm .


Note the following from the graph above:

- Horizontal and vertical scaling is identical, so true ribbon departure angles are portrayed by these spatial deflections.
- For the 5 cm wide case (heavy dotted line), the maximum deflection was attained simultaneously with peak wind; this max deflection then stayed constant at this value.
- Compare this to the 10 cm case, in which the maximum deflection occurred after peak wind was reached, with peak deflection continually growing for the entire period of constant peak wind.
- This is indicative of the conclusions presented in items 1-5 of section 4.2.

The two graphs below illustrate snapshots of horizontal and vertical air load density taken through the above response.



These air loads, plotted against ribbon arc-length, are consistent with the ribbon's becoming increasing horizontal as seen by the migration of peak air load along the ribbon length. This is because as a section of ribbon becomes progressively horizontal, it contributes ever less to horizontal and vertical air load; it is the area of ribbon curvature from horizontal-to-vertical where the air load action is occurring (see the section below for more detailed discussion of aerodynamic response).

This process is accompanied by very little tension change as shown the graph below; this graph is an envelope of tension snapshots along the length of the ribbon at various times covering the duration of the response. The profile is essentially that of the un-occupied, unperturbed elevator.


Below, it can be seen that stress has been little affected by the wind response.


### 4.3.2 Extreme Aerodynamic Loading

This section addresses the situation where the wind reaches a critical level that essentially transcends the ribbon displacement restoring mechanism, giving the wind almost full control over the ribbon, with almost unlimited ability to displace the ribbon horizontally. A useful analogy for understanding the existence of such a critical load and subsequent unlimited response might be termed the "Rolling Pin Analogy". Consider a rope hanging from the ceiling with a weight attached under gravity as shown below.


Imagining pushing on the rolling pin to deflect the rope from its initial vertical position. At any subsequent (steady) deflection, the applied force $\mathbf{F}$ (the rolling pin, or wind, in our analogy) must equilibrate both the vertical and the horizontal components of the weight $\mathbf{W}$. Note, all ribbon segments with a horizontal component of orientation is subject to generating vertical air load. Note that the weight manifests itself as a tension $\mathbf{W}$ acting on both sides of the rolling pin since the rolling pin functions as a pulley. It is readily seen that if the horizontal component of $\mathbf{F}$ exceeds that of $\mathbf{W}$ (between rolling pin and ceiling), then there is no limit to how far the rolling pin can be moved in the horizontal direction by $\mathbf{F}$ except for the energy available to the process creating $\mathbf{F}$. Likewise, if the vertical component of $\mathbf{F}$ exceeds W , then there is no limit to how far the weight $\mathbf{W}$ can be lifted (due to the geometry of horizontal deflection) except for the energy available to the process creating $\mathbf{F}$. The application of such an analogy to the elevator ribbon must first establish uniformity of tension around the bend of the characteristic curvature of the ribbon under air load (ie. the pulley assumption). This can be established by simulation, and further conceptually corroborated by noting these important facts:

1. Aerodynamic friction manifests virtually zero load tangent to the ribbon curvature (actually zero in the GTOSS aerodynamic model); thus a steady state tension differential cannot be sustained via aerodynamics.
2. The tension disturbance propagation speed of $28 \mathrm{~km} / \mathrm{sec}$ is so high compared to the rate of onset of air loads due to wind loading, that any tension disturbance gradients will quickly propagate away neutralize any disturbance source and attain gradient-free equilibrium.
3. The low effective spring rate of the ribbon allows it to easily tolerate elongations associated with being displaced downwind by hundreds of kilometers without significant increase in tension.

Now consider a response (shown in section 4.3.3.3) in which the above described critical aerodynamic load has apparently been reached. The resulting horizontal displacement is shown below.


The explanation for such response can be found in examining the air loads on the ribbon and relating that to the rolling pin analogy. In the above ribbon deflection snapshots, notice that the characteristic geometry of the ribbon transition zone from vertical to (more or less) horizontal, uniformly replicates itself from snapshot-tosnapshot, and presents a significant geometrical opportunity for vertical air load to be created. The pressure distribution along the length of the ribbon creates an air load density vector normal to the ribbon's tangent at each point. Integrating this spatial force distribution around this characteristic curvature results in both a horizontal and vertical component of net air load as shown in the diagram below illustrating how air loads distribute themselves along the ribbon.


Quantification of the above is provided by the air load density distribution snapshots below corresponding to this particular response (shown in section 4.3.3.3).



The two graphs above show snapshots of air load density versus length along the ribbon for the case of the climber at LEO subject to a category 0 wind; at any given point in time, the total vertical or horizontal air load on the ribbon is simply the "area under the curve" for the corresponding snapshot time (with due regard for the units of the ari load density).

It is significant that once the wind velocity attains a critical level, both components of air load density are migrating along the ribbon with their magnitudes as well as ratios almost unchanged (the dark solid-line snapshots corresponding to the period of constant peak wind). A progressive flattening phenomenon depends upon the air load being able to equilibrate both the horizontal and vertical components of ribbon tension beneath the climber. Since the ribbon bends from almost horizontal to almost vertical, and since tension is essentially constant over this region, this means that both the horizontal and vertical components of net air load must be nearly equal to the tension. Under these conditions, the ribbon exhibits a compliance to being blown downwind with little apparent resistance; in this case, the "area under the curve" (total horizontal or vertical air load) of the air load snapshots actually become equivalent to the tension in the ribbon.

It appears that once the wind-tension balance reaches a point that the wind can lay the ribbon horizontal, then there may be only insignificant natural restoring mechanism remaining since the energy available to the wind for "performing the work required to displace the ribbon horizontally" is essentially unlimited.

### 4.3.3 Typical Ribbon Deflection Response Shapes

The graphs in this section further and more dramatically illustrate the observations of section 4.2 above.

### 4.3.3.1 Deflections for un-occupied elevator

Below is the case of the unoccupied ribbon under a category 3 wind ( $54 \mathrm{~m} / \mathrm{s}=120 \mathrm{mph}$ )
The graph below shows snapshots of the entire length of the ribbon, including the time of constant peak-wind and tail-off; comparing category 3 wind response for an identical configuration (except) under a category 0 wind indicates a factor of 100 greater horizontal response for this category 3 wind. Note the horizontal scaling; while this response is significant, if it were viewed from a real vantage point that encompassed the entire ribbon length, it would appear essentially straight, with horizontal distortion on the order of $1 \%$ of its length.


The graph below shows magnified, near-earth, horizontal displacement snapshots with identical vertical and horizontal scaling to depict true geometry and ribbon
departure angles; also shown on this graph is the maximum deflection for the category 0 wind illustrating the overall effects of wind speed on ribbon departure angle.


Note the single heavy vertical dashed line nearest the origin (above); this represents the maximum horizontal response for a category 0 wind. It is evident that somewhere between category 0 and 3 wind levels, a threshold was reached for which wind force could overcome any inherent ribbon resistance to horizontal displacement. This supposition is further corroborated by the fact that the bracketed snapshots (with heaviest lines near the middle deflections of the graph), encompass exactly the period of constant peak wind, meaning that even with wind not increasing, the horizontal displacement continues to increase.

### 4.3.3.2 Deflections with climber in low atmosphere

Here, a 20 ton climber is parked in the atmosphere at 9 km , and subjected to the Category 0 wind profile. Note, climber aerodynamics were characterized as a simple drag model with drag coefficient of 1.2 and cross sectional area of $18 \mathrm{~m}^{2}$, thus only horizontal air load is generated by the climber.

Horizontal ribbon displacement is shown below. Note the one snapshot composed only of dots; this depicts GTOSS finite tether model nodal spacing below and above the climber, and shows a resolution below the climber that is just adequate to resolve these aerodynamics; this sparseness was adopted for numerical efficiency. Location of the climber is evidenced by the sharp bend at about 9 km along the ribbon. This bend, while not pronounced near the ribbon's initial vertical position, ends up clearly depicted at a horizontal distance near 9 km ; thus the trajectory of the climber becomes evident.


Due to the low tension between the ground and climber, there is little resistance initially to horizontal displacement of the climber; this is exacerbated even further by the additional atmospheric drag on the climber. So the climber and lower ribbon section easily move horizontally pulling the climber even lower; however, once the climber has moved a horizontal distance corresponding to its fixed position on the ribbon (of 9 km ), then any additional action by air loads to move the climber horizontally is met by the now nearly horizontal segment of ribbon between the climber and ground. This short segment, with an effective spring rate about 10,000 times greater than the ribbon above, can easily equilibrate any horizontal load with very little additional strain as shown below as a tension time history in the lower ribbon segment. This shows that the tension rises to meet the applied horizontal air load, thus effectively constraining the climber itself (but not the ribbon above) from additional horizontal motion. Once the lower ribbon becomes near horizontal, the situation then mimics the displacement, shape and departure angles of the unoccupied elevator, as witnessed by the fact that in this case, the ribbon above the climber exhibits about 5 km of maximum horizontal displacement beyond the climber; this compares closely with the shape and peak displacement of the unoccupied elevator under a category 0 wind. The case of a climber parked in the atmosphere ( 9 km ) more nearly mimics the unoccupied configuration than it does the case with the climber parked at $200 \mathrm{~nm}(150,000 \mathrm{~m})$.


### 4.3.3.3 Deflections with climber at LEO

Below are snapshots of the ribbon, at $2,000 \mathrm{sec}$ intervals for 10 hours. Subjected to identical winds, a ribbon with a climber at LEO ( 200 nm ) responds vastly different than an unoccupied ribbon. Note different horizontal scales between this and the unoccupied ribbon. Maximum atmospheric displacement for an unoccupied ribbon was about 6,000 meters; with a climber at LEO, the maximum displacement is 150,000 meters!


The difference in response can be attributed to the effect of climber mass that serves to modify response in the following two significant ways: (a) by presenting a significant inertia that affects ribbon excursions near the atmosphere, and (b) by creating a significant ribbon tension drop across itself (see graph below), thus presenting to the atmosphere, a ribbon under 4 times less tension than for the unoccupied ribbon. The low tension presents a much more compliant ribbon to the wind than that of the unoccupied ribbon. The tension discontinuity shown below occurs at the climber's position of 370 km altitude ( 200 nm ).


Snapshots below use identical vertical and horizontal axis scaling to depict actual ribbon departure geometry, and indicates a ribbon eventually departing the anchor at near horizontal. One snapshot depicts dots representing GTOSS nodal resolution.


Below is the same as the figure above, except with a much greater vertical scale to show the climber's position. Here, the sharp bend in the ribbon, not seen above due to its scale, clearly depicts the location and effect of the climber. Note also the snapshot, composed of only dots at the nodal points; this shows where the nodal resolution changes at the location of the climber.


The graph below has magnified but also identically-scaled vertical and horizontal axes to provide insight into the deflection mechanism in this case.


Note, for each snapshot above, the sum of the "vertical distance to the climber" plus the "horizontal distance to the anchor" is essentially constant, and equal to the initial vertical altitude of the climber. By the time the simulation has terminated, the climber has been displaced downward by about 140 km , accompanied by no significant tension increases. This is consistent with both the insignificant increase in upper ribbon strain corresponding to this amount of climber displacement, as well as the upper ribbon's low effective end-to-end spring rate. Using an upper ribbon spring rate of $0.04 \mathrm{~N} / \mathrm{m}$, this decrease in climber altitude corresponds to a tension increase of 5000 N (out of 200,000 N extant in the ribbon above the climber); the corresponding strain increase in the upper ribbon due to this displacement is only 0.14 percent.

### 4.4 RIBBON HORIZONTAL DISPLACEMENT

This shows horizontal ribbon displacement response to reference wind variations of:

1. Peak Wind value varying between Category 0 and 3
2. For Category 3 wind, the Duration of Peak Wind, varying between 1 and 4 hours 3. For Category 0 wind, Ribbon Width varying between 5 and 20 cm .

### 4.4.1 Horizontal Displacement vs Wind Speed (Unoccupied)

Round markers are simulation-run data points; dotted lines are interpolation.


Note the sharp break in the displacement response above; this is indeed the point at which total air load achieves a level capable of equilibrating (virtually simultaneously) both vertical and horizontal components of ribbon tension. At this point, the wind essentially gains control of the ribbon, and limitation to further displacement will either be at the whim of the weather, or, will await second-tier effects (due to extreme displacements) to start manifesting themselves. Shown below is the same graph as above, except with (a)
magnified scale limited to $0-30 \mathrm{~m} / \mathrm{s}$ wind speed, and (b) the corresponding departure angle is overlaid on the graph.

To aid in the interpretation of this phenomenon, the graph below shows (for lower wind speeds) is the relationship between departure angle and horizontal displacement.


### 4.4.2 Horizontal Displacement vs Wind Duration (Unoccupied)

Shown below is response to a Category 3 wind level for an unoccupied ribbon.
Round markers are simulation-run data points; dotted lines are interpolation.


Duration of Peak Wind (hours)

Note: a "duration of zero" here means that as soon as the wind rises to the Category 3 level, it immediately starts to recede with no dwell time at category 3 level. It is apparent that the displacement-duration relationship is essentially linear up until sometime after 2 hours wind duration at which point, secondary effects are starting to come into play to eventually attenuate the progression of downrange displacement. This could be slowly increasing ribbon tension, geometry changes due to downrange distance (such as earth curvature).

### 4.4.3 Horizontal Displacement vs Ribbon Width (Unoccupied)

The graph below shows the ribbon horizontal displacement sensitivity to ribbon width for an unoccupied elevator ribbon. This data corresponds to a Category 0 peak wind level.

Round markers are simulation-run data points; dotted lines are interpolation.


### 4.4.4 Horizontal Displacement vs Wind Speed (Climber in Atmosphere)

The graphs below correspond to a climber parked at 9 km altitude on a 5 cm SE ribbon subjected to various wind levels. The presence of the climber in the atmosphere exhibits both beneficial as well as detrimental effects.

Beneficial effects pertaining to horizontal displacement:

- The fact that the portion of the ribbon between the climber and the anchor point becomes quickly horizontal under aerodynamic response means that this very stiff horizontal section of ribbon easily equilibrates the horizontal loads that can be developed in the high aerodynamic pressure regions associated with lower altitudes. This effect clearly manifests itself in the
deflection -vs- wind speed graphs below when compared to the unoccupied ribbon (section 4.4.1 above).


Round markers are simulation-run data points; dotted lines are interpolation.


### 4.4.5 Horizontal Displacement vs Wind Speed (Climber at LEO)

The graphs below correspond to a climber parked at LEO altitude on a 5 cm SE ribbon subjected to various wind levels. The presence of the climber in the atmosphere exhibits detrimental effects regarding horizontal displacement.

Round markers are simulation-run data points; dotted lines are interpolation.


### 4.5 RIBBON DEPARTURE ANGLES

The departure angle of the ribbon is the angle that the ribbon makes with the "local vertical" reference, thus 90 degrees indicates a ribbon's proceeding straight up, vertically, from the anchor point. While this parameter is not indicative of over stressing, or other structural failure per se for the ribbon, it does have profound implication on anchor station design, and general operations in the presence of wind. These results may well speak for operational constraints of climber launches based on wind predictions for the anchor location. Due to the possibility of low-tohorizontal departure angles, additional design criteria may be levied on the rib on to tolerate sea water contamination. Additionally, practical response to the potential for low departure angles my dictate anchor station design that provides an actual ribbon attach point elevated considerably above sea level.

### 4.5.1 Departure Angle vs Wind Speed (Unoccupied)

Round markers are simulation-run data points; dotted lines are interpolation.


### 4.5.2 Departure Angle vs Ribbon Width (Unoccupied)

The graph below shows the ribbon departure angle sensitivity to ribbon width for an unoccupied elevator ribbon. This data corresponds to a Category 0 peak wind level.

Round markers are simulation-run data points; dotted lines are interpolation.


### 4.5.3 Departure Angle vs Wind Speed (Climber in Atmosphere)

The graphs below correspond to a climber parked at 9 km altitude on a 5 cm SE ribbon subjected to various wind levels. The presence of the climber in the atmosphere exhibits detrimental effects regarding ribbon departure angles.

Detrimental effects pertaining to ribbon departure angle:

- Due to the low tension between the climber and anchor point, and the aerodynamic drag of the climber itself, even winds less than category 0 will virtually lay the climber down creating a horizontal departure angle. This effect clearly manifests itself in the departure angle -vs- wind speed graphs below.

Round markers are simulation-run data points; dotted lines are interpolation.



### 4.5.4 Departure Angle vs Wind Speed (Climber at LEO)

The graphs below correspond to a climber parked at LEO altitude on a 5 cm SE ribbon subjected to various wind levels. The presence of the climber at LEO exhibits detrimental effects regarding ribbon departure angles.

Detrimental effects pertaining to ribbon departure angle:

- Due to the low tension between the climber and anchor point, even winds less than category 0 will virtually lay the ribbon down creating a horizontal departure angle. This effect clearly manifests itself in the departure angle -vs- wind speed graphs below.

Round markers are simulation-run data points; dotted lines are interpolation.



Point of Interest: It is evident that in both cases of a Climber occupying the ribbon (ie. at LEO and within the atmosphere), that the ribbon departure angle quickly goes to horizontal under wind levels of Category 0 , and likely at even lesser winds. This speaks for the possibility of elevator launch restrictions when appreciable winds are predicted.

### 4.6 STRESS RELATED TO WIND RESPONSE

By design, nominal unoccupied SE stress levels are near constant; the question arises whether and how much wind can magnify these stress levels. This section addresses this by showing graphs of stress profile vs ribbon length for various conditions.


### 4.6.2 Stress Profile vs Ribbon Width (Cat 0, Unoccupied)

Here, the ribbon width is being varied, all subject to a Category 0 wind level. The wind level follows this scenario:

- 2 hr rise time to Cat 0 level
- 2 hr Cat 0 level wind duration
- 2 hr recession to zero wind






### 4.6.3 Stress Profile vs Wind (Climber in Atmosphere)

Note the characteristic tension drop across the climber at 9 km , that manifests itself as the discontinuity at the 9 km point on the ribbon. The ribbon above the climber is characteristically showing very little stress increase due to aerodynamic displacement, thus the horizontal axis is scaled to show only the first 40 km of ribbon. It is the short section of ribbon below the climber

(between climber and anchor) that is showing significant stress response. This can be explained as follows. First note that these graphs consist of a series of snapshots of stress profile along the ribbon at various points in time throughout the entire 6 hours of active wind. All these profiles exhibit identical snapshots during the period that the wind is insignificant, such as initially before it has built to peak values (this is the regime that is manifesting itself as the lowest stress values being experienced in the region between ground and 9 km ). As wind level increases with time, the stress snapshots in this region also start to increase. This is because as wind increases, the climber is being laid down ever more horizontal, thus the short ribbon section below the climber is functioning to equilibrate the total horizontal air loads on the ribbon. While normally, the tension in the ribbon below the climber exhibits a lower value than that above, in this case, as the ribbon becomes horizontal, its tension starts to assume the full level required to equilibrate the horizontal component of air load; this component of the process essentially imbues the lower ribbon (originally under reduced stress) with stress levels consistent with an unoccupied elevator under air load. Now, added to this increasing tension is the air load that arises from the fact that the climber itself has aerodynamic drag subject to lower altitude high atmospheric density. All this combines to create a condition of significant stress increase in the lower ribbon for the case of the climber in the atmosphere.

Note, that the stress above 9 km shows almost no increase due to the extremely low effective spring rate of the ribbon above the climber as described in preceding sections.

### 4.6.4 Stress Profile vs Wind (Climber at LEO)

This section shows stress level response for an elevator with a climber parked on the ribbon at LEO, or $200 \mathrm{~nm}(370 \mathrm{~km})$.


Note that similar tendencies are being exhibited in this case as that for the case of the climber parked in the atmosphere, except, here stress amplification is greatly mitigated over the atmospheric case. The section of ribbon between anchor and climber exhibits a much smaller effective spring rate than that for the atmospheric climber. Note that as peak wind levels increase the ribbon is being stretched (seen in the migration of the climber's position to greater distances from the ground, ie. moving towards the right in the series of snapshot graphs).

Stress profiles over the remainder of the ribbon are typical and show no specific tendency toward increased stress.

Note, results for the Category 3 wind were curtailed due to limitations on run time during analysis, so the response tendencies being logically established as wind peak progressed through category $0,1,2$, are not fully developed for the category 3 wind.

